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Holography in general space-times

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Abstract: We provide a background-independent formulation of the holographic principle. It permits the construction of embedded hypersurfaces (screens) on which the entire bulk information can be stored at a density of no more than one bit per Planck area. Screens are constructed explicitly for AdS, Minkowski, and de Sitter spaces with and without black holes, and for cosmological solutions. The properties of screens provide clues about the character of a manifestly holographic theory.

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1. Holographic Principle

1.1 The holographic principle for general spacetimes

In ref. [1] a covariant entropy bound was conjectured to hold in general space-times. The bound can be saturated, but not exceeded, in cosmology and in collapsing regions. Applied to finite systems of limited self-gravity, it reduces to Bekenstein's bound [2]. For a D-dimensional Lorentzian space-time, the covariant bound can be stated as follows:

Covariant entropy conjecture *Let A be the area of a connected $(D-2)$-dimensional spatial surface B. Let L be a hypersurface bounded by B and generated by one* *of the four null congruences orthogonal to B. Let S be the total entropy contained on L. If the expansion of the congruence is non-positive (measured in the direction away from B) at every point on L, then $S \leq A/4$.*

The conjecture can be viewed as a generalization of an entropy bound proposed by Fischler and Susskind [3]. It differs in that it considers all four light-like directions and selects some of them by the criterion of non-positive expansion. Several concepts crucial to a light-like formulation were recognized earlier by Corley and Jacobson [20], who distinguished between "past and future screen maps" and drew attention to the role of the expansion. We should also point out a number of recent proposals for entropy bounds in cosmology [4, 5, 6, 7, 8] (see sec. 3.4).

The covariant entropy bound is manifestly invariant under time reversal. This property cannot be understood if the bound applies only to thermodynamic entropy. One is thus forced to interpret the bound as a limit on the number of degrees of freedom ($N_{\rm dof}$) that constitute the statistical origin of any thermodynamic entropy that may be present on L. Since no assumptions about the microscopic properties of matter were made, the limit is fundamental [1]. There simply cannot be more independent degrees of freedom on L than $A/4$, in Planck units. This conclusion compels us to embrace the holographic conjecture of 't Hooft [9] and Susskind [10], and it motivates the following background-independent formulation of their hypothesis:

Holographic principle *Let A be the area of a connected $(D-2)$-dimensional spatial surface B. Let L be a hypersurface bounded by B and generated by one of the four null congruences orthogonal to B. Let $\mathcal{N}$ be the number of elements of an orthonormal basis of the quantum Hilbert space that fully describes all physics on L. If the expansion of the congruence is non-positive (measured in the direction away from B) at every point on L, then $\mathcal{N} \leq e^{A/4}$.*

Simplifying slightly,[1] one could state that $N_{\rm dof} \leq A/4$, *where $N_{\rm dof}$ is the total number of independent quantum degrees of freedom present on L.*

The holographic principle thus assigns at least two light-like hypersurfaces to any given spatial surface B and bounds $N_{\rm dof}$ on those hypersurfaces. The relevant hypersurfaces can be constructed as follows (see ref. [1] for a more detailed discussion). There will be four families of light-rays orthogonal to B: a past-directed and a future-directed family on each side of B. Consider only the families with non-positive expansion away from B. Generically there will be two such families, but if the expansion is zero in some directions, there may be as many as three or four. Pick one of the allowed families and follow each light-ray until the expansion becomes positive or a boundary of space-time is reached. The null hypersurface thus generated is called a *light-sheet*. $N_{\rm dof}$ on a light-sheet of B will not exceed a quarter of the area of B.

[1]We thank Gerard 't Hooft for suggesting a formulation in terms of Hilbert space.

In general, the holographic principle associates the area of a surface B with N_{dof} on null hypersurfaces, not spatial regions, bounded by B. Under certain conditions, however, the bound does apply to space-like hypersurfaces as well. This was shown in ref. [1] for the covariant entropy bound. For the holographic principle, the derivation can be repeated, with "entropy" replaced by "N_{dof}." It yields the following theorem:

Spacelike projection theorem *Let A be the area of a closed surface B possessing a future-directed light-sheet L with no boundary other than B. Let the spatial region V be contained in the intersection of the causal past of L with any spacelike hypersurface containing B. Let N_{dof} be the total number of independent quantum degrees of freedom present on V. Then $N_{\mathrm{dof}} \leq A/4$.*

The theorem can be widely applied and easily understood. Under the stated conditions, none of the degrees of freedom on V can escape through holes in L or be destroyed on a singularity. Then causality and the second law of thermodynamics require all degrees of freedom on V to be present on L as well. On L their number is bounded by the holographic principle, so it must be bounded also on V. The spacelike projection theorem will be of use in a number of spacetimes, including de Sitter and AdS (sec. 3).

1.2 Outline

The holographic principle is a relation between space-time geometry and the number of degrees of freedom. It is not equivalent to the statement that there exists a conventional theory without gravity, living on the boundary of a space-time region, with one degree of freedom per Planck area, by which all bulk phenomena including quantum gravity can be described. The holographic principle is clearly necessary for the existence of such a theory, but as we will argue below, it is not sufficient.

The holographic principle does imply, however, that all information contained on L can be stored on the surface B, at a density of no more than one bit per Planck area. (We neglect factors of $\ln 2$.) We shall work with this interpretation. For a given spacetime, we ask whether the information in the interior can be completely projected (in accordance with our formulation of the holographic principle) onto suitable hypersurfaces which will be called screens. We are led to a construction (sec. 2) under which the space-time is sliced into null hypersurfaces L. Each light-ray on L is followed in the direction of non-negative expansion until the expansion becomes zero. This yields a preferred location for a screen encoding the information on L. By repeating this procedure for every slice L, one obtains one or more screen-hypersurfaces. They will either be located on the boundary or will be embedded in the interior of the spacetime. We establish conditions under which projection along spacelike directions is possible.

In sec. 3 we apply the construction to examples of space-times, including anti-de Sitter space, Minkowski space, de Sitter space, cosmological solutions, and black holes. We find that AdS has highly special properties under our construction, as it

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admits spacelike projection onto timelike screens of constant area. For all examples we find that the information in the entire spacetime can be projected onto preferred screens. For spaces with black holes, inequivalent slicings lead to different screen structures. We relate this to the question of information loss.

We discuss the structure of holographic screens in sec. 4, and draw some conclusions. In a number of examples, the screen-hypersurfaces are spacelike. In other examples they are null or timelike, with the spatial area depending on time. One would not expect such screens to admit a conventional Lorentzian quantum field theory with one degree of freedom per Planck area, because the number of degrees of freedom would have to be time-dependent.

This suggests that a distinction should be made between a *dual theory*, and the *holographic theory* (sec. 4.2). In both types, the holographic principle would be manifest. A dual theory would be characteristic of a certain class of space-times. It would be a conventional theory without gravity, living on the geometric background defined by a holographic screen of the space-time and containing one degree of freedom per Planck area. It would complement, or be equivalent to, a quantum gravity theory living in the bulk, and could thus be used to describe bulk physics. The conformal field theory on the boundary of AdS [11, 12, 13, 14] is an example of a dual theory.

The existence of a dual theory in a given class of space-times will depend on certain properties of the projection and the screen which we aim to expose. More generally, the structure of screens points to a fundamental theory in which quantum degrees of freedom are a derived concept, and their number can change. The theory must give rise to gravity by permitting the unique reconstruction of space-time geometry from the effective number of degrees of freedom in such a way that the holographic principle is manifestly satisfied (sec. 4.3). We call this the holographic theory. Clearly it cannot be a conventional quantum field theory living on a pre-defined geometric background. Perhaps an indication of its character can be gained from certain proposals of 't Hooft [9, 15].

Notation and conventions We work with D-dimensional Lorentzian manifolds M. The terms *light-like* and *null* are used interchangeably. Any $(D-1)$-dimensional submanifold $H \subset M$ is called a *hypersurface* of M [16]. If $D-2$ of its dimensions are everywhere spacelike and the remaining dimension is everywhere timelike (null, spacelike), H is called a *timelike (null, spacelike) hypersurface*. By a *surface* we always refer to a $(D-2)$-dimensional spacelike submanifold $B \subset M$; by *area* we mean the proper volume of a surface. By a *light-ray* we do not mean an actual electromagnetic wave or photon, but simply a null geodesic. We use the terms *null congruence*, and *family of light-rays*, to refer to a congruence of null geodesics [16, 17]. The term *light-sheet* is defined in sec. 1.1; the terms *projection*, *screen*, and *screen-hypersurface* in sec. 2.1; *dual theory* and *holographic theory* in sec. 4.2. We set $\hbar = c = G = k = 1$.

2. Holographic projection

2.1 Screens

The construction described in the previous section answers the following question: Given a surface B of area A, what is the hypersurface L on which $N_{\rm dof}$ is bounded by $A/4$? We will now consider space-times globally, and ask a different question: which surfaces store the information contained in the entire space-time? To answer this question, the above prescription should be inverted. Given a null hypersurface L, one should follow the geodesic generators of L in the direction of non-negative expansion. One can stop anytime, but one must stop when the expansion becomes negative. This procedure will be called *projection*. The $(D-2)$-dimensional spatial surface B spanned by the points where the projection is terminated will be called a *screen* of the projection. If the expansion vanishes on every point of B, it will be called a *preferred screen*.

Preferred screens are of particular interest for a simple reason. The expansion of the projection typically changes sign on a preferred screen B. Therefore B will be a preferred screen for projections coming from two directions, e.g., the past-directed outgoing and future-directed ingoing directions. It will thus be particularly efficient in encoding global information. (Actually, there may be a deeper reason why preferred screens play a special role. We suspect that they are precisely the surfaces for which the holographic bound, $N_{\rm dof} \leq A/4$, is saturated. This is suggested by considerations in sec. 4.2 of ref. [1]. There it was found that the covariant entropy bound can be saturated on the future light-sheet of an apparent horizon, but not on the future light-sheets of smaller spheres inside the apparent horizon. Under the projection that generates those light-sheets, the apparent horizon is a preferred screen. This argument should be viewed with caution, however, because there might be independent, practical reasons why the thermodynamic entropy cannot be made as large as $N_{\rm dof}$ for the smaller spheres.)

By following all generators of the null hypersurface L in a non-contracting direction to a screen, we obtain a projection of all information on the hypersurface onto one or more screens, which may be embedded in the hypersurface, or may lie on its boundary. The number of screens can be minimized by using preferred screens whenever possible.

2.2 Screen-hypersurfaces

In order to project the information in a space-time M, our strategy will be to slice M into a one-parameter family of null hypersurfaces, $\{L\}$. This will be possible in all examples we consider. Usually the slicing is highly non-unique, but the symmetries of most space-times of interest reduce the number of inequivalent slicings considerably. To each slice L, we apply the projection rule. This procedure yields a number of one-parameter families of $(D-2)$-dimensional screens. Each family forms a $(D-1)$-

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dimensional screen-hypersurface embedded in M or located on the boundary of M. (This sounds a lot more complicated than it is — see the "recipe" in sec. 2.3 and the figures in sec. 3 below.) The screen-hypersurfaces can be time-like, null, or space-like; in sec. 3 examples of each type will be found. In general, the causal character can change from time-like to space-like within the screen-hypersurface.

Usually it will be clear whether we are talking about a screen (a spatial surface), or a $(D-1)$-dimensional hypersurface formed by a one-parameter family of screens. Therefore we will often refer to a screen-hypersurface loosely as a "screen" of M. If the hypersurface consists of preferred screens, we call it a preferred screen-hypersurface, or loosely a preferred screen of M. If the expansions of both independent pairs of orthogonal families of light-rays vanish on a screen, it will be preferred under all four projections that end on it. We will call such a screen, and hypersurfaces formed by such screens, *optimal.*

So far we have discussed only *null projection*, i.e., projection of information along null hypersurfaces. It is sometimes possible to project information along spacelike hypersurfaces. Namely, *spacelike projection* of the information in a spatial region V onto a screen B is allowed if V and B satisfy the conditions set forth in the "spacelike projection theorem" (sec. 1.1). This will be significant in a number of space-times, in particular in de Sitter and anti-de Sitter space.

2.3 The recipe

To construct screens, one must slice a space-time into null hypersurfaces. In view of the spherical symmetry of all metrics considered below, it will be natural to slice them into a family $\{L\}$ of light-cones centered at $r = 0$.[2] The family can be parametrized by time. This will leave two inequivalent null projections, namely along past or future-directed light-cones. Often the light-cones will be truncated by boundaries of the space-time and will not include $r = 0$, but this does not matter. In the case of spherical symmetry, one thus obtains the following recipe for the construction of screens:

1. Draw a Penrose diagram. Every point represents a $(D-2)$-sphere. Each diagonal line represents a light-cone. The two inequivalent null slicings can be represented by the ascending and descending families of diagonal lines.

2. Pick one of the two families. Now the question is in which direction to project along the diagonal lines.

3. Identify the apparent horizons, i.e. hypersurfaces on which the expansion of the past or future light-cones vanishes. They will divide the space-time into normal, trapped, and anti-trapped regions. In each region, draw a wedge whose legs point in the direction of negative expansion of the cones.

[2]In Minkowski-space, we will also consider a family of light-rays orthogonal to a flat $(D-2)$-plane.

4. On a given diagonal line (i.e. light-cone), project each point towards the tip of the local wedge, onto the nearest point (i.e. sphere) B_i where the direction of the tip flips, or onto the boundary of space-time as the case may be.

5. Repeat for every line in the family. The surfaces B_i will form (preferred) screen-hypersurfaces H_i.

Below we will strive to make these steps explicit by including two or three Penrose diagrams for most examples. In the first diagram, the apparent horizons will be identified and the wedges placed. For each inequivalent family of light-cones, we will then provide a diagram in which the projection directions are indicated by thick arrows. We invite the reader to verify that these directions are uniquely determined by the wedges. Screens will be denoted by thick points, preferred screen-hypersurfaces by thick lines.

3. Examples

3.1 Anti-de Sitter space

Type IIB string theory on the background $AdS_5 \times \mathbf{S}^5$, with N units of flux on the $\mathbf{S}^5$, appears to be dual to $(3+1)$-dimensional $\mathrm{U}(N)$ supersymmetric Yang-Mills theory with 16 real supercharges [11]. One can consider this theory to live on the boundary of the AdS space. The correspondence between bulk and boundary [12, 13] relates infrared effects in the bulk to ultraviolet effects on the boundary [18, 19, 13]. This feature was exploited by Susskind and Witten [14] to show that the boundary theory has only one degree of freedom per Planck area, as required by the holographic principle in the traditional, "spacelike" form in which it has often been expressed.

We wish to understand some of these properties from the perspective of the general formulation of the holographic principle given in sec. 1.1. From this point of view, the bulk information is projected along null directions in general, and along spacelike directions only if certain conditions are met. We will verify that these conditions are indeed satisfied in AdS. Moreover, we will find that the boundary at spatial infinity is a preferred (and optimal) screen under our construction. Finally, we will note that AdS admits screen-hypersurfaces of constant spatial area that encode their space-time interior. The concurrence of these properties is special to AdS (and to some unstable solutions identified in sec. 3.5), and may be a necessary condition for the existence of the kind of duality that has been found in this space-time.

Anti-de Sitter space can be scaled into the direct product of an infinite time axis with a unit spatial ball [14]. In this form it has the metric

$$ds^2 = R^2\left[-\frac{1+r^2}{1-r^2}dt^2 + \frac{4}{(1-r^2)^2}\left(dr^2 + r^2 d\Omega^2\right)\right]. \tag{3.1}$$

The constant scale factor R is the radius of curvature. The spacelike hypersurfaces are open balls given by $t = \text{const}, 0 \leq r < 1$. The boundary of space is a two-sphere residing at $r = 1$. The proper area of spheres diverges as $r \to 1$.

Consider the past directed radial light-rays emanating from a caustic ($\theta = +\infty$) at $r = 0, t = t_0$ (fig. 1).

They form a past light-cone L with a spherical boundary. The cone grows with affine time until the light-rays reach the boundary of space at $r = 1$. It is straightforward to check that the expansion, θ, is inversely proportional to the affine time. It thus decreases monotonically, but remains positive; one finds that

$$\theta \to 0 \qquad \text{as} \qquad r \to 1\,. \tag{3.2}$$

Consider a sphere B, of area A_B, on the lightcone L. The part of L in the interior of B, L_B, has negative expansion in the direction away from B, and therefore constitutes a light-sheet of B. By the holographic principle, the number of degrees of freedom on L_B does not exceed a quarter of the area of B:

$$N_{\rm dof}(L_B) \leq \frac{A_B}{4}\,.$$

Because the cone closes off at $r = 0$, it has no boundary other than B. The spatial interior of B on any spacelike hypersurface through B, V_B, lies entirely in the causal past of L_B. Therefore the conditions for the spacelike projection theorem are met. It follows that area bounds $N_{\rm dof}$ on spatial regions of AdS:

$$N_{\rm dof}(V_B) \leq \frac{A_B}{4}\,.$$

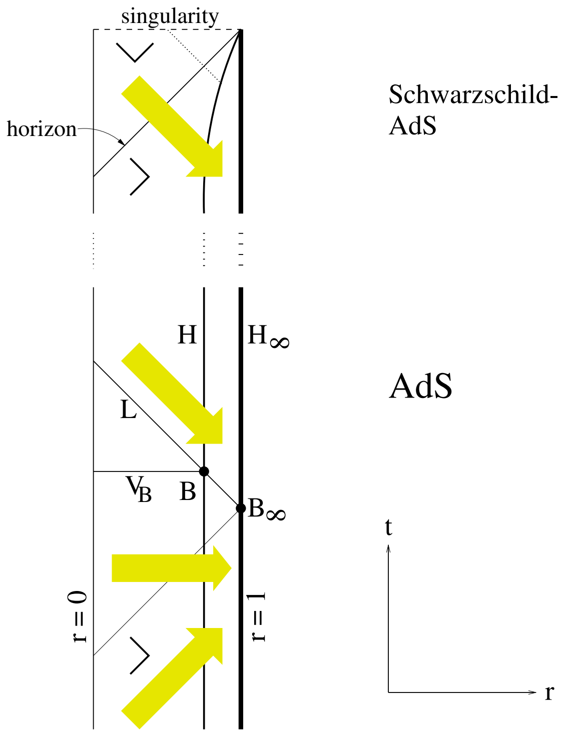

Figure 1: Conventions and methods used in all diagrams are spelled out in sec. 2.3. Anti-de Sitter space contains no apparent horizons; all spheres are normal. Spacelike projection is allowed. All null and spacelike projections are directed away from the center at $r = 0$. Interior information can thus be projected onto a screen-hypersurfaces H of constant area; H encodes no exterior information. The screen at spatial infinity, H_∞, is optimal and encodes all bulk information. — The upper part of the figure shows a diagram for Schwarzschild-AdS. Since the future light-sheets of the screen surfaces are not complete (see dotted line), the black hole interior cannot be projected onto H along space-like directions, but only along past light-cones.

Since the light-cone expansion is positive for all values of r, these conclusions remain valid in the limit as the sphere B moves to the boundary of space, $B \to B_\infty$.

By eq. (3.2), B_∞ is a preferred screen. As expected, the preferred screen is precisely the one which encodes the entire space. By time reversal invariance of eq. (3.1), the expansion of future-directed radial lightrays arriving at B_∞ will also vanish; thus B_∞ is an optimal screen.

So far we have considered screens bounding $N_{\rm dof}$ on a particular light-cone or spatial hypersurface. A screen-hypersurface encoding the entire space-time is obtained by repeating the construction for every single light-cone in the slicing of AdS. By the time-translation invariance of eq. (3.1), this repetition is trivial. The family of finite screens $B(t)$ of constant area A_B thus forms a timelike screen-hypersurface H of topology $\mathbb{R} \times \mathbf{S}^{D-2}$. By the holographic principle, $N_{\rm dof}$ in the enclosed space-time region does not exceed $A_B/4$. From the spacelike projection theorem it follows that it does not matter whether one counts degrees of freedom on null or on spacelike hypersurfaces intersecting H. After taking the limit $B(t) \to B_\infty(t)$, one finds that the timelike boundary at $r = 1$, H_∞, is an optimal screen of anti-de Sitter space. It encodes the entire information in the bulk, by spacelike or null projection.

Let us briefly discuss what happens when a black hole forms. This is shown in the upper part of fig. 1. Let us assume that the constant screen area, A_B, is so large that the black hole never engulfs the screen-hypersurface H formed by the screens $B(t)$. Then the past-directed ingoing light-sheet of any $B(t)$ lies outside the event horizon, and has no boundary other than $B(t)$. By arguments similar to those leading to the spacelike projection theorem [1], this implies that $N_{\rm dof}$ in the region between the black hole and H never exceeds $A_B/4$. Generic space-like hypersurfaces passing through the interior of the black hole, however, are not contained in the causal past of any complete future-directed light-sheet of B (see dotted line in fig. 1). The spacelike projection theorem does not apply to those regions, and therefore the spacelike projection of the interior of the event horizon onto H is not possible. Of course, the entire black hole interior can be encoded on H by null projection along past light-cones. (Alternatively, it can be projected onto the apparent horizon; we discuss this in more detail in sec. 3.2 for the case of Schwarzschild black holes.) This discussion remains valid in the limit as $B \to B_\infty$, and thus applies to the boundary of Schwarzschild-AdS at spatial infinity.

3.2 Minkowski space

We now turn to space-times which are asymptotically flat. The discussion of finite bound systems in Minkowski space does not differ much from the treatment in AdS. The space-time region occupied by them can be projected onto a screen-hypersurface of topology $\mathbb{R} \times \mathbf{S}^{D-2}$, formed by a spherical screen circumscribing the system. As long as no black holes form, the projection can be spacelike. A spherical screen of finite size is not preferred unless the interior is on the verge of gravitational collapse (see sec. 3.5).

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A bound system can also be projected along past-directed light-rays onto a remote flat plane. All families of null-geodesics orthogonal to the plane have zero expansion; therefore the screen is optimal. It can encode bulk information on both sides. This projection was originally proposed by Susskind [10] and was further investigated in ref. [20]. If the system does not contain black holes, it can be projected onto the plane along future-directed light-rays as well.

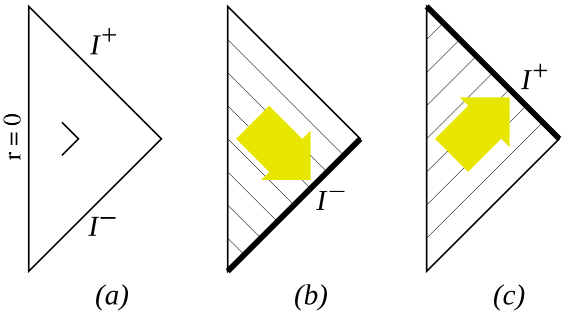

Figure 2: Like AdS, Minkowski space contains no trapped or anti-trapped spheres (a). Unlike AdS, the two allowed null projections lead to different preferred screens, I^- (b) and I^+ (c). Either screen is sufficient to encode the entire spacetime. This can be viewed as an expression of the unitarity of the S-matrix.

For the discussion of scattering processes (fig. 2) we shall follow the recipe given in sec. 2.3.

By following past light-cones centered at $r = 0$, all of Minkowski space is projected onto past null infinity, I^-, where $\theta \to 0$. Similarly, by following future light-cones one can project the bulk onto future null infinity, I^+. Both infinities are preferred screens of Minkowski space. Each screen alone suffices to store all information in the interior of the space-time; one can interpret this as a statement of the unitarity of the S-matrix [21] in the absence of black holes.

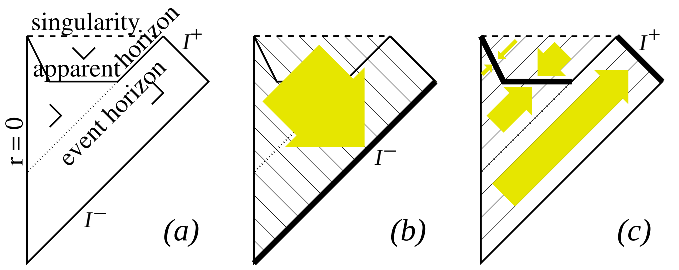

Figure 3: A classical black hole forms in a scattering process. The spheres within the apparent horizon are trapped (a). All information can be projected along past light-cones onto I^- (b). But I^+ only encodes the information outside the black hole; this reflects the information loss in the classical black hole. The black hole interior can be projected onto the apparent horizon (c).

Let us now assume that a black hole forms during scattering (fig. 3a).

The past light-cones still project all points in the spacetime onto I^-, including the interior of the black hole (fig. 3b). The screen I^+, however, encodes only the exterior of the black hole, via future-directed outgoing light-rays (fig. 3c). This discrepancy can be interpreted as information loss in classical black holes. The black hole interior can be encoded onto the apparent horizon, which forms a preferred screen, by future-directed outgoing and past-directed ingoing lightrays.

The picture becomes more interesting when the quantum radiation of black holes, as well as its back-reaction, is included. This restores the possibility of unitarity. After the black hole has formed from classical matter, the apparent horizon shrinks due to the quantum pair creation of particles [22]. In this process a positive energy particle escapes to infinity, while its negative energy partner crosses into the black hole. Unlike positive energy matter, this particle anti-focusses light: it violates the null convergence condition and causes the expansion of light-rays to increase. Outgoing light-rays immediately inside the horizon can thus change from negative to positive expansion without going through a caustic (fig. 4).

(If the null convergence condition [16] holds, the expansion becomes positive only at "caustics," or focal points, of the light-rays. Caustics thus are the generic endpoints of light-sheets [1].) This leads to a situation which would not be possible in a classical space-time. There exists a hypersurface H, namely the black hole apparent horizon during evaporation, from which one has to project *away* in *both* directions. Thus, past-directed ingoing light-rays map H onto a different part of the apparent horizon (h), and future-directed outgoing light-rays map it onto a part of I^+.

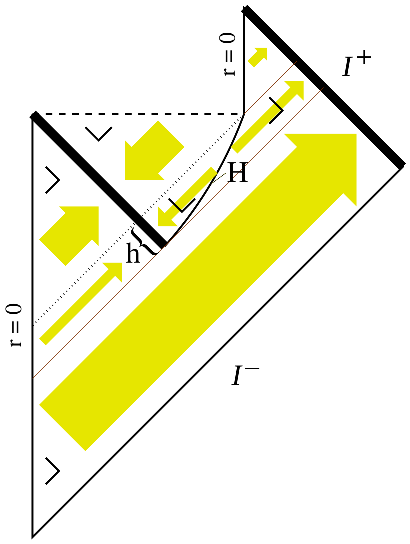

Figure 4: A quantum black hole forms in a scattering process. Because negative energy particles cross the apparent horizon during the evaporating phase (H), its size decreases. The expansion of future light-cones immediately inside the apparent horizon changes from negative to positive in this process. Therefore the maximal area of the apparent horizon marginally exceeds the maximal area of the event horizon. The diagram shows the projection of this space-time along future light-cones onto screens formed by the apparent horizon and by I^+ (thick lines). The past light-cones would lead to the usual projection onto I^-.

A digression on unitarity. Let us examine the evaporation process in more detail. When a negative mass particle enters the horizon, the expansion of the generators of the horizon changes from zero to a positive value. There will be a nearby null congruence, inside the black hole, whose expansion is changed from a negative value to zero by the same process. This congruence will now generate the apparent horizon. Since it has smaller cross-sectional area, the horizon has shrunk. The movement of the apparent horizon will leave behind a trace in the Hawking radiation, causing a deviation from a thermal spectrum over and above the deviation caused by greybody factors. This is similar to the distortion in the thermal spectrum of radiation enclosed in a cavity, while the wall of the cavity is being moved.

The amount by which the apparent horizon decreases during a given pair-creation event depends on the *profile*, $\theta(\mathcal{A})$, of the expansion of the outgoing future-directed null geodesics near the horizon. The cross-sectional area $\mathcal{A}$ of null congruences becomes smaller, and the expansion θ more negative, the further inside the black hole they are located. If the black hole was formed by a system of low entropy, for example by the collapse of a homogeneous dust ball of zero temperature, the profile will be a featureless monotonic function, and the horizon will decrease very smoothly during evaporation. The back-reaction, in this case, will not imprint a significant signature onto the thermal spectrum. However, if the black hole was formed by a highly enthropic system, the profile will be more complex.

Consider a shell of matter falling into a black hole. For now, assume that the shell contains only radial modes, and is thus exactly spherically symmetric even microscopically. If a lot of entropy is stored in the shell, its density will be a complicated function of the radius. By Raychauduri's equation [16, 17], the density profile of the infalling shell will be imprinted on the expansion profile of the outgoing future-directed null geodesics that eventually pass through the apparent horizon during evaporation. Correspondingly, the same type of pair creation process will sometimes cause the horizon area to decrease by a larger step, sometimes by a smaller amount, depending on the expansion profile of the null geodesics passing through H at the pair creation event. The back-reaction will be irregular, and the corresponding deviations of the Hawking radiation from the thermal spectrum will be complex. There is thus a signature in the radiation which encodes the irregularity of the back-reaction, which in turn encodes the complexity of the matter system that formed the black hole.

It is easy to extend this discussion to systems containing also angular modes. They will deflect outgoing lightrays into angular directions. The expansion will now be a local function of the cross-sectional area, $\theta(\delta\mathcal{A};\vartheta,\varphi)$. The back-reaction will not be spherically symmetric, and the apparent horizon will develop dents and bulges. This leaves a non-spherical signature in the Hawking radiation.

In this way information about the material falling into the black hole may be transferred onto the outgoing Hawking radiation. The information will be encoded in a subtle way and it will typically be necessary to measure the entire radiation emitted by the black hole before the ingoing state can be reconstructed. Of course, we have sketched only a qualitative picture, and we have taken the pair creation model of black hole evaporation rather literally. Moreover, no mechanism can copy ingoing information onto outgoing radiation unless one implicitly assumes that the fundamental theory evades the "quantum Xeroxing" no-go theorem [23], for example by non-locality [24].[3] We have aimed to outline a specific mechanism by which information may be transferred in the semi-classical picture. In general terms, our discussion is strongly related to the approach of 't Hooft [25, 26, 27].

3.3 de Sitter space

de Sitter space is the maximally symmetric solution of the vacuum Einstein equation with a positive cosmological constant Λ. It may be visualized as a $(D-1,1)$-

[3] We thank Lenny Susskind for pointing this out, and for a number of related discussions.

hyperboloid embedded in $(D+1)$-dimensional Minkowski space. A metric covering the entire space-time is given by

$$ds^2 = -dt^2 + H^{-2} \cosh^2 Ht \;\; d\Omega_{D-1}^2 \,, \tag{3.3}$$

where

$$H = \sqrt{\frac{\Lambda}{3}} \tag{3.4}$$

is the Hubble parameter, or inverse curvature radius. In this metric, the spacelike hypersurfaces are spheres, $\mathbf{S}^{D-1}$. They contract, and then expand, at an exponential rate. de Sitter space also admits metrics with maximally symmetric spatial sections of zero or negative curvature, as well as a static metric,

$$ds^2 = -(1 - H^2 r^2) d\tau^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega_{D-2}^2 \,. \tag{3.5}$$

Those metrics cover only certain portions of the spacetime.

The metric on the spatial $(D-1)$-sphere is given by:

$$d\Omega_{D-1}^2 = d\chi^2 + \sin^2\chi \; d\Omega_{D-2}^2 \,, \tag{3.6}$$

whence

$$r = H^{-1} \cosh Ht \;\, \sin\chi \,. \tag{3.7}$$

A geodesic observer is immersed in a bath of thermal radiation [28] of temperature $T = H/(2\pi)$, which appears to come from the cosmological horizon surrounding the observer. The causal structure of de Sitter space is shown in fig. 5.

The only boundaries are past and future infinity, I^- and I^+; they are both spacelike. The space-time is divided in half by the event horizon, E, of a geodesic observer, who can be taken to live at $\chi = r = 0$.

Consider the past light-cones centered at $\chi = 0$, i.e. at one of the two poles of the $\mathbf{S}^{D-1}$. Just as for AdS and Minkowski, the expansion starts with the value $+\infty$ and decreases. Any surface on the light-cone bounds $N_{\rm dof}$ on the part of the cone it encloses. By the spacelike projection theorem, it also bounds $N_{\rm dof}$ on any spatial hypersurface in its interior. Perhaps surprisingly, this holds even for surfaces which are both near the event horizon and near the past singularity (see fig. 5a, dashed line). Their area will be $\sim H^{-2}$, but they enclose an exponentially large spatial region.

When the light-cone reaches I^-, the expansion approaches zero. Thus the boundary surface on past infinity is a preferred screen. (Actually it is optimal because a past light-cone arriving from the other pole, $\chi = \pi$, will also have $\theta \to 0$ near I^-.) Repeating this projection for all times, one finds that half of de Sitter space, namely the region within the event horizon of an observer at $\chi = 0$, can be projected onto past infinity. The projection of only half of the space-time is peculiar to de Sitter space. By contrast, past light-cones project all of Minkowski, or all of AdS, onto their respective infinities (secs. 3.1 and 3.2). By using past light-cones emanating from the

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other pole of the spatial $\mathbf{S}^{D-1}$, an additional portion of de Sitter can be projected onto past infinity. But this still leaves out the antitrapped region beyond the cosmological horizons. It can only be projected by future-directed lightrays onto future infinity. A global null projection of de Sitter space can thus be achieved by using two optimal screen-hypersurfaces: past and future infinity. This is shown in fig. 5*b*. Indeed, the potential holographic role of these boundaries has been speculated upon for some time [21]. Both global screens are spacelike hypersurfaces. Because surfaces near future infinity are anti-trapped, one cannot encode global de Sitter space on a finite number of timelike or null screens.

However, we can apply the spacelike projection theorem (sec. 1.1) to the screens that form the event horizon E of an observer at $\chi = 0$. They are all of constant area, $4\pi H^{-2}$. (Since E never reaches $\chi = r = 0$, this argument strictly requires a limiting procedure starting from spheres in the vicinity of E; see fig. 5*a*.) Because E is generated by light-rays of zero expansion, all screens on it are manifestly preferred. Thus, the null hypersurface E is a preferred screen of constant area. Because of its degeneracy, E encodes only itself under null projection along E. Under spacelike projection, however, it encodes half of the space-time (fig. 5*c*), namely the region within the event horizon. One can reasonably argue that the region beyond the event horizon has no meaning because it cannot be observed, and that de Sitter space should not be treated globally [29, 30] (fig. 5*c*); thus the screen E should suffice for a holographic description of de Sitter space.

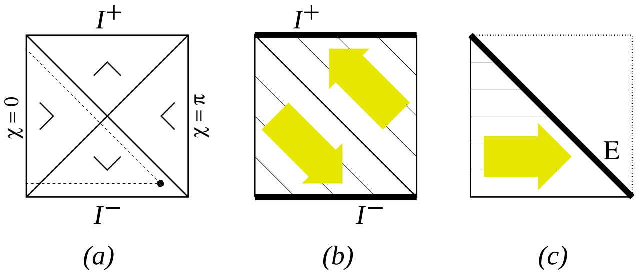

Figure 5: Penrose diagram for de Sitter space. The $\mathbf{S}^{D-1}$ spacelike slices would correspond to horizontal lines through the square. The diagonals are apparent horizons dividing the space-time into four regions (a). The $(D-2)$-spheres near past (future) infinity are trapped (anti-trapped); the spheres near the poles are normal. Null projection must be directed towards the tips of the wedges (see sec. 2.3). It follows that de Sitter space can be projected onto past and future infinity (b), which are spacelike, optimal screen-hypersurfaces of (exponentially) infinite size. A more interesting screen is obtained by applying the spacelike projection theorem to spheres near the event horizon E of an observer at $\chi = 0$ (a). By taking a limit, one can show that all information in the observable region of de Sitter space can be projected onto the preferred screen E, which is a null hypersurface of constant spatial area $4\pi H^{-2}$ (c).

In inflationary models, the de Sitter phase is followed by a matter or radiation dominated phase, and the entire space-time during this era can be projected onto the screens available in the relevant FRW models [1] (see sec. 3.4). — Black holes in de Sitter space can be treated much like black holes in AdS or Minkowski space.

3.4 FRW cosmologies

Friedmann-Robertson-Walker (FRW) cosmologies are described by a metric of the form

$$ds^2 = a^2(\eta)\left[-d\eta^2 + d\chi^2 + f^2(\chi)d\Omega^2\right]. \tag{3.8}$$

Here $f(\chi) = \sinh\chi$, χ, $\sin\chi$ corresponds to open, flat, and closed universes respectively. FRW universes contain homogeneous, isotropic spacelike slices of constant (negative, zero, or positive) curvature. We will not discuss open universes, since they display no significant features beyond those arising in the treatment of closed or flat universes.

The matter content will be described by $T_{ab} = \mathrm{diag}(\rho, p, p, p)$, with pressure $p = \gamma\rho$. We assume that $\rho \geq 0$ and $-1/3 < \gamma \leq 1$. The case $\gamma = -1$ corresponds to de Sitter space, which was discussed in sec. 3.3. The *apparent horizon* is defined geometrically as the spheres on which at least one pair of orthogonal null congruences have zero expansion. It is given by

$$\eta = q\chi\,, \tag{3.9}$$

where

$$q = \frac{2}{1+3\gamma}\,. \tag{3.10}$$

The solution for a flat universe is given by

$$a(\eta) = \left(\frac{\eta}{q}\right)^q. \tag{3.11}$$

Its causal structure is shown in fig. 6.

The interior of the apparent horizon, $\eta \geq q\chi$, can be projected along past light-cones centered at $\chi = 0$, or by space-like projection, onto the apparent horizon. The exterior, $\eta \leq q\chi$, can be projected by the same light-cones, but in the opposite direction, onto the apparent horizon. The apparent horizon is thus a preferred screen encoding the entire space-time. Alternatively, one can use future light-cones to project the entire universe onto future null infinity, another preferred screen.

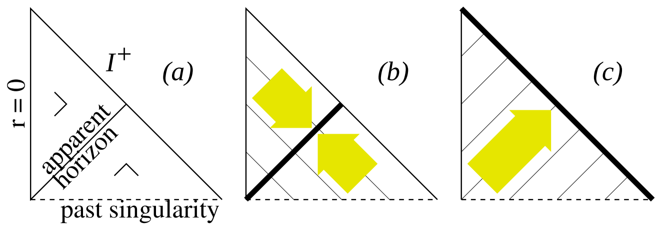

Figure 6: Penrose diagram for a flat FRW universe dominated by radiation. The apparent horizon, $\eta = \chi$, divides the space-time into a normal and an anti-trapped region (a). The information contained in the universe can be projected along past light-cones onto the apparent horizon (b), or along future light-cones onto null infinity (c). Both are preferred screen-hypersurfaces.

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By eq. (3.9), the apparent horizon screen is a timelike hypersurface for $-1/3 < \gamma < 1/3$, null for $\gamma = 1/3$, and spacelike for $1/3 < \gamma \leq 1$. In a universe dominated by different types of matter in different eras, the causal character of the apparent horizon hypersurface can change from timelike to spacelike or vice versa (see, e.g., fig. 5 in ref. [1]).

For a closed universe, the solution is given by

$$a(\eta) = a_{\max} \left(\sin \frac{\eta}{q} \right)^q . \tag{3.12}$$

In addition to eq. (3.9), a second apparent horizon emanates from the opposite pole of the spatial $\mathbf{S}^{D-1}$, at $\chi = \pi$; it is described by

$$\eta = q(\pi - \chi) . \tag{3.13}$$

The two hypersurfaces formed by the apparent horizons divide the space-time into four regions, as shown in fig. 7.

Let us choose the first apparent horizon, eq. (3.9), as a (preferred) screen-hypersurface. On one side, $\eta \geq q\chi$, lies a normal region and a trapped region. These regions can be projected onto the screen by past-directed radial light-rays moving away from the South pole ($\chi = 0$). The other half of the universe, $\eta \leq q\chi$, can be projected onto the same screen by future directed radial light-rays moving away from the North pole ($\chi = \pi$). Therefore the preferred screen given by eq. (3.9) encodes the entire closed universe.

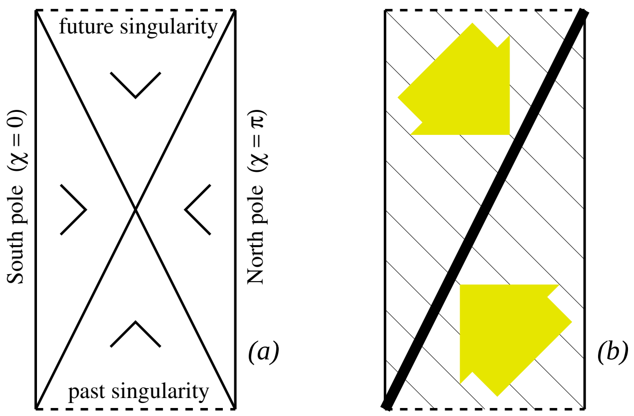

Figure 7: Penrose diagram for a closed FRW universe dominated by pressureless dust. Two apparent horizons divide the space-time into four regions (a). The information in the universe can be projected onto the embedded screen-hypersurface formed by either horizon (b).

A number of cosmological entropy bounds have been proposed [4, 5, 6, 7, 8] which are based on the idea of defining a horizon-size spatial region to which Bekenstein's bound can be directly applied. We have emphasized the importance of these bounds in ref. [1], where we also discuss their relation to the covariant entropy bound (sec. 1.1). Those bounds can be given a holographic interpretation by considering them as limits on N_{dof} in the specified ken. Because they refer to limited regions, however, it is not clear how global screen-hypersurfaces could be constructed.

3.5 Einstein static universe

The Einstein static universe (ESU) is a closed FRW space-time containing ordinary matter as well as a positive cosmological constant of a certain critical value [16, 31]. Its metric can be written as a direct product of an infinite time axis with a $(D-1)$-sphere of constant radius a:

$$ds^2 = -dt^2 + a^2 d\Omega_{D-1}^2 \,. \tag{3.14}$$

The causal structure is shown in fig. 8.

Each hemisphere can be projected along past- or future-directed light-rays, or by spacelike projection, onto the equator. This screen is optimal, because all four families of orthogonal light-rays have vanishing expansion. Moreover, the screen forms a timelike hypersurface, with spatial slices of constant finite size.

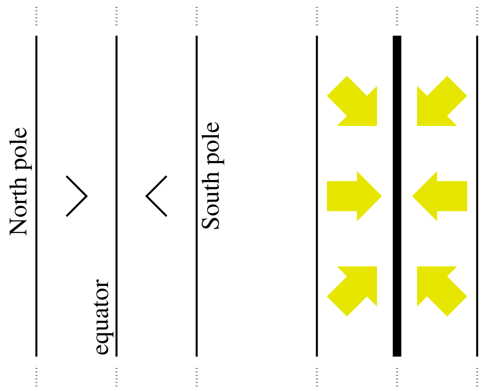

Figure 8: Penrose diagram for the Einstein static universe. The equator separates two normal regions. It forms an optimal, timelike screen-hypersurface of constant area, encoding all information by null or spacelike projection. These properties are shared by the boundary of AdS.

This is reminiscent of the properties of the screen at the boundary of anti-de Sitter space: the screen is optimal, timelike, of constant size, and encodes the entire space-time by spacelike projection. The difference is that the AdS screen has infinite proper area, while the equator in the ESU is a $(D-2)$-sphere of finite area $\sim a^{D-2}$. It lies not on a boundary of space (there is none in the ESU), but is embedded in the interior.

The properties of the projection might give rise to the hope that a boundary theory, dual to the bulk description, could be formulated on the equator of the ESU. The example may be of limited use, however, because the ESU is not a stable solution [16, 31]. Another unstable solution with similar properties is given by a static spherical system just on the verge of gravitational collapse. Its radius will be equal to its gravitational radius, and the expansion of both past- and future-directed outgoing light-rays goes to zero at the surface of the system.

4. Holographic theory

4.1 Summary

From a universal entropy bound found in ref. [1], we obtained a background-independent formulation of the holographic principle [9, 10]. This led us to a construction of hypersurfaces (screens) on which all information contained in a space-time can be stored. The screens are embedded, or lie on the boundary of the space-time, and contain no more than one bit of information per Planck area. In this sense, the world is a hologram.

The construction was applied to a number of examples. For anti-de Sitter space it yields the timelike boundary at spatial infinity as a preferred screen. In Minkowski space, past or future null infinity, or a flat plane, can encode all information. de Sitter space is mapped along light-rays onto the spacelike infinities in the past and future; alternatively, all information in the observable half of the de Sitter space can be stored on the event horizon (a null hypersurface of finite area) via spacelike projection. Cosmological spacetimes may not have a boundary, but embedded screens can be found; they may be spacelike, timelike, or null, depending on the matter content. The information in a black hole can be mapped onto the apparent horizon, or onto past null infinity.

From the examples one can draw the following observations:

- Holographic screens can be spacelike hypersurfaces.
- If they are timelike or null, the spatial area is not necessarily constant in the induced, or any other, time-slicing.

Before explaining why these features may be significant, let us briefly discuss a tempting but misguided conclusion. One might argue that holographic projection onto spacelike screens is a trivial accomplishment, because in any conventional theory one can specify initial conditions on a Cauchy surface and predict, or retrodict, the past and future development of the system. That is true, but it is a different kind of information storage. In that case, one stores not only the information at one moment of time, but also a machine (namely the theory) which is capable of recovering the state of the system at all other times. A holographic construction, on the other hand, feigns ignorance of any theory describing the matter evolution, and simply encodes all information, at all times, onto screens of dimension $D-2$. The space-time is sliced into null hypersurfaces; slice by slice the information is encoded onto $(D-2)$-dimensional spatial surfaces at a maximum density of one bit per Planck area. These surfaces form a $(D-1)$ dimensional screen hypersurface which may be timelike, spacelike, or null, but from the point of view of holographic information storage its causal character is irrelevant.

4.2 Theories on the screen

Our interpretation, so far, has centered on the *information* needed to describe a state. This is measured by the number of degrees of freedom. Therefore we can use the holographic principle (which refers to $N_{\rm dof}$) to project all information in the space-time onto screen-hypersurfaces. In this sense, the holographic principle implies a drastic reduction of the complexity of nature compared to naive expectations of, perhaps, one degree of freedom per Planck volume.

We did not, however, use the holographic principle to *describe* nature. The holographic principle is far from manifest in the description of the world in terms of general relativity and quantum field theory; yet these theories are very successful. Working within their frame, one finds a number of non-trivial effects which appear to insure that the entropy bound implied by the holographic principle is always satisfied [1]; but these results could not have been immediately inferred from the basic axioms of GR and QFT.

As a kind of external restriction imposed on physical theories, holography is interesting but unsatisfactory. If the number of degrees of freedom is limited by the holographic principle, there ought to be a description of nature in which this restriction is manifest. Let us call this hypothetical description *the holographic theory*. One would expect the holographic theory to remain valid when semi-classical gravity breaks down [9, 11]; in this regime it may be the only possible description. These are good reasons to search for a holographic theory.

The simplest idea would be to define a theory on the geometric background given by the screen-hypersurface(s). If the theory contained one degree of freedom per Planck area, and was related by a kind of dictionary ("duality") to the space-time ("bulk") physics, the holographic principle would be manifest. Let us call this type of theory a *dual theory*.

This idea works for certain asymptotically anti-de Sitter space-times. The screen encoding the entire bulk information is the timelike hypersurface formed by the boundary of space (sec. 3.1). According to Maldacena's remarkable conjecture [11], a super-Yang-Mills theory living on this hypersurface describes the bulk physics completely. By considering a finite boundary and taking the limit as it moves to spatial infinity, one can show that the theory contains no more than one degree of freedom per Planck area [14]. Therefore it is a dual theory in the sense of our definition.

Perhaps it will be possible to find dual theories for some other classes of space-times; certainly this would be be an important contribution to the understanding of holography and of quantum gravity. In general, however, the "dual theory" approach will not work. The theories we usually think of have a fixed number of degrees of freedom built into them; these degrees of freedom evolve in Lorentzian time. But consider the cosmological solutions studied in sec. 3.4. The area of the screens is time-dependent. The screen theory would have to be capable of "creating" or "activating" degrees of freedom. Moreover, the area can decrease, as seen in the closed universe example. In the screen theory this would correspond to the destruction, or de-activation, of degrees of freedom. Eventually their number would approach zero, and the second law of thermodynamics would be violated in the screen theory.[4] (Note that this does not, of course, imply a violation of the second law in the bulk. Rather,

[4] We are grateful to Andrei Linde for stressing this point to us.

it is related to the creation, or destruction, of degrees of freedom at the initial and final singularities of the universe.)

This suggests that one should not in general think of the screen theory as a conventional theory with a fixed number of degrees of freedom. Rather, one might expect it to be a theory with a varying number of "active" degrees of freedom. Thus, its properties would be very different from those of ordinary physical theories. Moreover, since the screen hypersurfaces can be spacelike or null, one should not expect the theory to live in Lorentzian time.

4.3 Geometry from entropy

We would like to advocate a more radical approach. One should not be thinking about a "screen theory" (a theory defined on some hypersurface of space-time) at all. The screen theory approach cannot be fundamental, because it presumes the existence of a space-time background, or at least of an asymptotic structure of space and time. In order to use the holographic principle for a full description of nature, we suggest it should be turned around. Loosely speaking, one should not constrain entropy by geometry, but construct geometry from entropy. (Strictly, "number of degrees of freedom" should replace "entropy" here.) The construction must be such that the holographic principle, in the form given in sec. 1.1, is automatically satisfied.

It thus appears that two problems must be overcome if a holographic theory is to be found. First, one must formulate a theory with a varying number of degrees of freedom. A possibility may be that the theory can activate or de-activate degrees of freedom from an infinite reservoir.[5] An extreme but perhaps more satisfying resolution would be to treat quantum degrees of freedom not as fundamental ingredients, but as a derived concept. 't Hooft has long been advocating that models should be sought in which quantum degrees of freedom arise as a complex, effective structures (see ref. [15] and references therein). It would be natural for $N_{\rm dof}$ to vary in such models.

The second challenge is to find a prescription that allows the unique reconstruction of space-time geometry from the varying number of degrees of freedom (see ref. [32] and references therein for a discussion of related questions). Part of this prescription will be to equate $N_{\rm dof}$ with the proper area of an embedded $(D-2)$-dimensional preferred or optimal screen. A more difficult question is how the intrinsic geometry of screen-hypersurfaces can be recovered. It may be undesirable to identify the discrete steps of, say, a cellular automaton [33, 34] with Lorentzian time. But the number and character of the degrees of freedom provide information about the matter content. Therefore a complete reconstruction of space-time geometry is not inconceivable.

[5]We thank Lenny Susskind for this suggestion and related discussions.

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